LAST Name: ANSWERS

First Name: _____

Student Number: _____

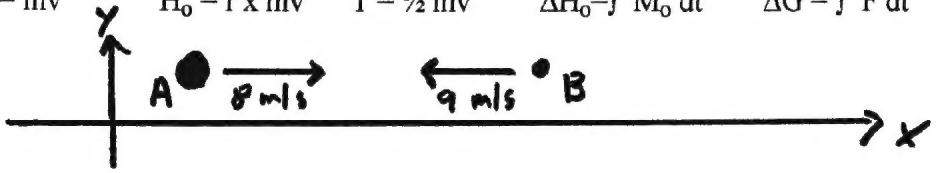
MIE 200F - Quiz number 5a – October 16, 2002
quiz duration = 20 minutes

Sphere "A" has a mass of 6 kg, and an initial velocity of 8 m/s i. Sphere "B" has a mass of 4 kg, and an initial velocity of -9 m/s $\hat{i}$. When the two spheres bounce off each other, 200 joules of heat are generated. Following the collision, the velocity of sphere "B" is vertical, in the +ve $\hat{j}$ direction. Ignore the effects of gravity.

3 (a) What is the x-component of the velocity of sphere "A" after the collision?

7 (b) What is the speed of sphere "B" after the collision?

$T_2 = T_1 + U_{1\to 2}$ $\quad \Sigma \vec{M}_o = \vec{H}_o$ $\quad \vec{G} = m\vec{v}$ $\quad \vec{H}_o = \vec{r} \times m\vec{v}$ $\quad T = \frac{1}{2} mv^2$ $\quad \Delta\vec{H}_o = \int \vec{M}_o \, dt$ $\quad \Delta\vec{G} = \int \vec{F} \, dt$



(a) No external forces on system of 2 particles

$\Rightarrow \Delta\vec{G} = 0$ in collision.

$$G_{1x} = G_{2x}$$

$$(6\,kg)(8\,m/s) + (4\,kg)(-9\,m/s) = (6\,kg)(V_{A,x}) + 0$$

$$V_{A,x} = 2\ m/s$$

(b) $T_2 = T_1 + U_{1\to 2}$ – energy lost in collision

$$\tfrac{1}{2} m_A \left(V_{A,x}^2 + V_{A,y}^2\right)_{final} + \tfrac{1}{2} m_B \left(V_{Bx}^2 + V_{By}^2\right)_{final} = \tfrac{1}{2} m_A (8^2) + \tfrac{1}{2} m_B (9)^2 - 200$$

but $(V_{A,x})_{final} = 2$

$(V_{B,x})_{final} = 0$

$V_{A,y\,final} = -\frac{2}{3} V_{By\,final}$ (conserve y momentum)

$$\Rightarrow 3\left(2^2 + \left(\tfrac{2}{3} V_{By}\right)^2\right) + 2\left(0 + V_{By}^2\right) = 3\times 64 + 2\times 81 - 200$$

$$12 + 3.33\, V_{By}^2 = 154 \Rightarrow \vec{V}_{By} = 6.5\ m/s\ \hat{j}$$

office 6(a)

LAST Name: Answer

First Name: _____

Student Number: _____

MIE 200F - Quiz number 5b – October 18, 2000
quiz duration = 20 minutes

(Based on homework problem 3/232)

The two spheres of equal mass m = 2 kg are able to slide *with zero friction* along the horizontal rotating rod. They are initially latched in position a distance r = 3 meters from the rotating axis., with the assembly rotating freely with an angular velocity ω = 10 *revolutions per second.* Neglect the small mass of the rod and shaft.

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(a) Find the initial angular momentum of the system with respect to the central axis.

(b) Find the initial linear momentum of the system with respect to the ground.

(c) Find the angular momentum of sphere "P" with respect to sphere "Q".

(d) At time t = 0, the latching mechanism releases the two masses, allowing them to slide out towards the ends of the rods. Find the *speed* of each of the two masses, just before they crash into the two ends of the rod.

$\vec{a} = \dot{v}\,\hat{e}_t + v\dot{\theta}\,\hat{e}_n = \dot{v}\,\hat{e}_t + v^2/\rho\,\hat{e}_n$ $\quad \vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$ $\quad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$

$T_2 = T_1 + U_{1\to 2}$ $\quad \Sigma \vec{M}_o = \vec{H}_o$ $\quad \vec{G} = m\vec{v}$ $\quad \vec{H}_o = \vec{r} \times m\vec{v}$ $\quad T = \tfrac{1}{2} mv^2$ $\quad \Delta \vec{H}_o = \int \vec{M}_o\, dt$

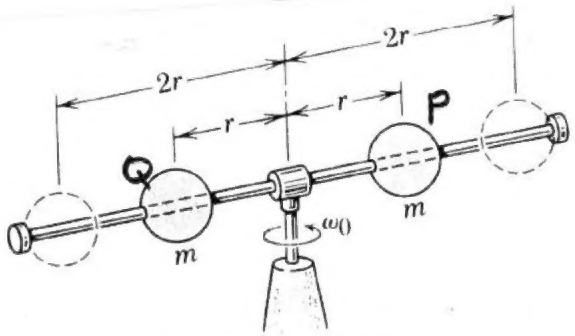
(a) $\vec{H}_o = 2\,\vec{r} \times m\vec{v}$

$= (2)(3)(2)(\omega r)$

but $\omega = 2\pi)(10 s^{-1}) = 62.8 s^{-1}$

$\Rightarrow \vec{H}_o = (2)(3)(2)(62.8)(3)$

$= 2261\ kg m^2/s$



Problem 3/232

(b) 2 masses have opposite velocities

$\Rightarrow \vec{G} = 0$

(c) $H_o = \vec{r} \times m\vec{v}$

$= (6)(2)(\omega r)$

$= (6)(2)(62.8)(6)$

$= 4522\ kg m^2/s$

(d) Up to point of impact, there have been no forces acting on spheres during the sliding motion

$\Rightarrow$ no work is done

$\Rightarrow T, |\vec{v}|$ are unchanged $|\vec{v}| = \omega r|_{initial}$

$= (62.8)(3) = 188.4\ m/s$

office 6(b)

LAST Name: ANSWERS

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MIE 200F - Quiz number 5b – October 17, 2002
quiz duration = 20 minutes

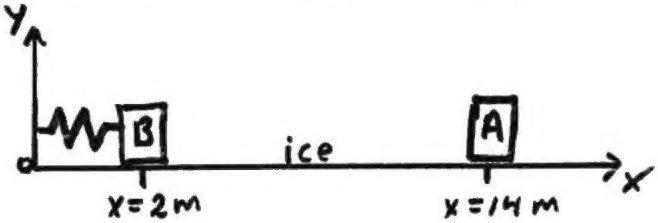
Block "A" has a mass of 4 kg, and block "B" has a mass of 10 kg. "A" is initially at rest, and "B" has an initial velocity of 8 m/s $\hat{i}$ at the position shown. One end of the spring is tied to "B"; the other end is anchored to the ground. The spring has a relaxed length of 9 meters, and a spring constant of 10 N/m. When the two blocks collide, they stick together and move in the positive $\hat{i}$ direction. Assume the ice exerts zero frictional force.

4 (a) What is the speed of "B" when it collides with "A"?

6 (b) What percentage of the kinetic energy is lost from the system of two blocks in the collision? Assume that the collision takes place over an extremely short time span.

$T_2 = T_1 + U_{1\to 2}$ $\quad \Sigma M_o = H_o$ $\quad G = mv$ $\quad T = \frac{1}{2} mv^2$

$\Delta H_o = \int M_o \, dt$ $\quad \Delta G = \int F \, dt$ $\quad U_{spring} = -\frac{1}{2} k (x_2^2 - x_1^2)$



(a) For "B" sliding on ice, $T_2 = T_1 + U_{1\to 2}$

$\frac{1}{2} m_B (V_{B,2})^2 = \frac{1}{2} m_B (V_{B,1})^2 - \frac{1}{2} k (x_2^2 - x_1^2)$

$(\frac{1}{2})(10)(V_{B,2})^2 = (\frac{1}{2})(10)(8^2) - (\frac{1}{2})(10)(5^2 - (-7)^2)$

$5 (V_{B,2})^2 = 320 + 120 = 440 \Rightarrow V_{B,2} = 9.4 \text{ m/s}$

(b) During collision, no external forces in x-direction

$\Rightarrow G_{1x} = G_{2x}$ $\quad m_B V_{B,1} = (m_A + m_B)(V_{AB,2})$

$\Rightarrow \frac{V_{AB,2}}{V_{B,1}} = \frac{m_B}{m_A + m_B} = \frac{10}{10+4} = 0.714$

% kinetic energy lost $= \left(1 - \frac{T_2}{T_1}\right) \times 100\% = \left(1 - \frac{\frac{1}{2}(m_A + m_B)(V_{AB,2})^2}{\frac{1}{2} m_B (V_{B,1})^2}\right) \times 100\%$

$= \left(1 - \frac{(.714)^2}{.714}\right) \times 100\% = 28.6\%$

office 6(c)

LAST Name: ANSWER

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Student Number: _____

MIE 200F - Quiz number 5a – October 17, 2000
quiz duration = 20 minutes

(Based on homework problem 3/241)

The small particle of mass m = 5 kg and its restraining cord are spinning on the horizontal surface of a smooth disc. A sectioned side view is shown in the Figure below. Initially, r = 3 meters and is constant. The initial angular velocity of the cord and mass is $\omega = 4\ s^{-1}$.

2 (a) Find the initial angular momentum of the mass with respect to the center of the disc.

3 (b) Find the force F needed to maintain the distance r at 3 meters.

5 (c) The force F is very slowly increased, until the distance r is reduced to 2 meters. How much work is done by the force F to accomplish this task?

$\vec{a} = \dot{v}\,\hat{e}_t + v\dot{\theta}\,\hat{e}_n = \dot{v}\,\hat{e}_t + v^2/\rho\,\hat{e}_n$ $\quad \vec{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$ $\quad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\,\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{e}_\theta$

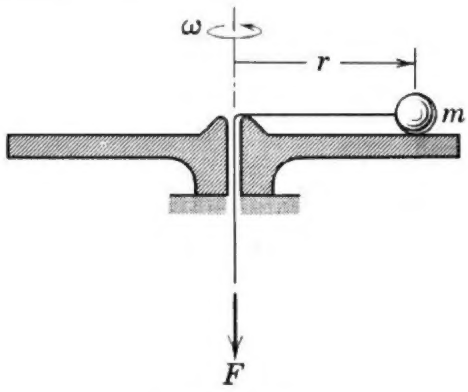
$T_2 = T_1 + U_{1\to 2}$ $\quad \Sigma \vec{M}_o = \dot{\vec{H}}_o$ $\quad \vec{H}_o = \vec{r} \times m\vec{v}$ $\quad T = \tfrac{1}{2} mv^2$ $\quad \Delta\vec{H}_o = \int \vec{M}_o\, dt$

(a) $\vec{H}_o = \vec{r} \times m\vec{v}$

but $\vec{r}$ is perpendicular to $\vec{v}$

$\Rightarrow |\vec{H}_o| = |\vec{r}|(m)(\omega|\vec{r}|)$

$\vec{H}_o = (3)(5)(4)(3) = 180\ kgm^2/s$



Problem 3/241

(b) $F_n = ma_n = m\dfrac{v^2}{r} = \dfrac{m(\omega^2 r^2)}{r}$

$= m\omega^2 r = (5)(4)^2(3)$

$F = 240\ N$

(c) $\Delta H = \int M_o dt = 0 \Rightarrow$ final $H_o = 180$

$180 = r\,mv|^{final} = (2)(5)(v_{final}) \Rightarrow v_{final} = 18\ m/s$

$180 = r\,mv|^{initial} = (3)(5)(v_{initial}) \Rightarrow v_{initial} = 12\ m/s$

$U_{1\to 2} = T_2 - T_1 = \tfrac{1}{2} mv_2^2 - \tfrac{1}{2} mv_1^2 = (\tfrac{1}{2})(5)[18^2 - 12^2]$

$= 450$ Joules